Evolution of the modern photon[a)]

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The term "photon" represents at least four distinct models and carries different connotations for students and for practicing physicists. This reflects the long and complex historical evolution of the concept and its association with the largely misinterpreted principle of duality. The unsatisfactory nature of the corpuscular and wave packet models is discussed, and the pedagogical desirability urged of replacing them with a semiclassical approach in elementary presentations. Derivations of the photoelectric (PE) effect without photons are cited and a vector analysis is given, demonstrating that the PE effect cannot be considered as simply the interaction of a photon and electron.

I. INTRODUCTION

The status of quantum mechanics today is similar to that of calculus in the 18th century: Its successes are universally conceded, but its logical underpinnings continue to provoke dissension.[1] It has been remarked that "nobody understands quantum mechanics,"[2] and the list of famous critics of the dominant Copenhagen interpretation (albeit on different grounds) includes Einstein, Schrödinger, de Broglie, Landé, and von Laue.

One of the most puzzling features of standard quantum mechanics, especially to undergraduate students, has been the wave–particle duality. The duality of light, coupled with the corpuscular photon model, has been given many conflicting interpretations and has promoted almost universal confusion among nonexperts. In this article, we will attempt to trace the history and evolution of the photon concept and the rise of duality to demonstrate the anachronistic nature of the pre-QED models and the fact that they provide only illusory simplification at the cost of subsequent difficulties, and hence to show that they do not deserve continued prominence in elementary treatments.

II. EARLY HISTORY

In 1905, perhaps influenced by the success of the discrete (and believed to be charge-quantized) electron, Einstein had published a paper in which he suggested the utility of the *heuristic* [Einstein's term] concept of independent energy quanta in explaining Stokes' law of fluorescence, Planck's blackbody radiation formula, and the photoelectric effect.[3] In particular, on the basis of his model, Einstein was able to account for the fact that, as shown by Lenard, the number of photoelectrons ejected was proportional to the intensity of the incident light, while the velocity distribution was independent of that quantity. He also predicted that the maximum kinetic energy of the photoelectron should be a linear function of the incident frequency, regardless of the nature of the emitting substance, a deduction that was not conclusively verified for another decade. Einstein's lightquantum (*Lichtquanten*) hypothesis did not receive wide acceptance from other physicists at the time. The general reaction is well summed up by the recommendation for the appointment of Einstein to the Prussian Academy of Sciences submitted in 1913 by Planck *et al.*, which stated, in part, "That [Einstein] may occasionally have missed the mark in his speculations, as, for example, with his hypothesis of lightquanta, ought not to be held too much against him, for it is impossible to introduce new ideas, even in the exact sciences, without taking risk."[4]

Von Laue opposed the quanta and suggested that the quantization resided in matter, not radiation. For, in a situation reminiscent of Franklin's overthrow of the two-fluid theory of electricity, it was logically unnecessary to suppose that both matter and radiation should be quantized in order to account for the quantization of energy transfer between the two, while the fact that the interaction was only quantized for the case of bound matter was highly suggestive as to the source of the quantization.[5,6] (As even Einstein once remarked, "Although beer is sold only in pint bottles, it does not follow that it exists only in indivisible pint portions.")[7]

Compton's paper in 1923 gave a great impetus to acceptance of the photon model and has been termed a "turning point in physics."[8] Interestingly enough, Compton clung to classical theory and subsequently tried to reconcile the wave and particle models by likening an electromagnetic wave to "the sheet of rain that one sees sweeping down the street or across the fields. The radiation particles or photons would correspond to the rain drops of which the sheet is composed."[9]

Einstein had always employed the term "quantum" in his articles, but, in 1926, the American chemist G. N. Lewis published a paper claiming a law of conservation of quanta and coined the word "photon" in deliberate imitation of labels such as proton or electron.[10] Nothing has survived of this theory except the term, but the shift to a concrete image was significant.

Bohr rejected the existence of quanta for many years. In 1923, he had referred to the "insuperable difficulties" of the lightquantum hypothesis in accounting for interference phenomena and asserted that "the picture ... which lies at the foundation of the hypothesis of lightquanta ... excludes, in principle, the possibility of a rational definition of the conception of a frequency which plays a principal part in this theory."[11] Troubled by the advent of Compton's paper, Bohr welcomed the visiting Slater's notion of a virtual radiation field determining probabilities of atomic transitions. But, unlike the latter, he was unwilling to accept the real existence of quanta and, characteristically, insisted that the concept should be referred to only in ambiguous language.[12]

The resulting 1924 Bohr–Kramers–Slater paper attempted to dispose of the quanta by treating the interaction of radiation and matter as satisfying conservation laws only statistically in the time average. The theory was eventually refuted on experimental grounds, and, in 1927, Bohr finally accepted the photon at the price of espousing complementarity[13] and, thus, enshrining the wave–particle duality of light as a permanent feature of the Copenhagen interpretation, a denouement that has been described as "a case in which competing views, found unresolvable, were simply combined—perhaps a singular event in the development of science"[14] and one that it must be said has caused dissension ever since (see, e.g., Park[15]).

Bohr's conversion was decisive for acceptance of the photon model, which in one form or another has remained an article of faith in the mainstream physics community to this day. Contrary to the general belief that physics theories rise or fall solely on the basis of physical evidence, it would appear that the evidence is often conflicting and that a good deal of the short-run direction of physics, at least, has been determined by the judgments and philosophic preferences of strong personalities[16] (i.e., authorities) in the field (e.g., Lorentz,[17] Einstein, Bohr,[18] Eddington[19]).

III. THE EVIDENCE REVISITED

Einstein's original idea of the quanta was apparently suggested by the mathematical identity between the equations for entropy of blackbody radiation and entropy of an ideal gas.[20] Formal similarities of this sort have often been productive of fruitful new insights in physics, although they cannot in themselves be taken as sufficient evidence for physical correspondence. What evidence, then, is there for the existence of the historic photon, ignoring for the moment its many versions?

The Compton effect and the photoelectric effect are the phenomena most frequently cited in textbooks as evidence for the photon. However, neither is solely dependent on elementary photon models for its explanation, unless the fact that energy and momentum transfer between light and matter is proportional to the frequency, with proportionality constants h and h/c, respectively, is taken as sufficient evidence.

The Compton shift is given by

$$\Delta\lambda = (h/m_e c)(1 - \cos\theta) \qquad (1)$$

for $h\nu_0 \gg E$, the electron binding energy, as will be assumed hereafter. Small discrepancies reported in experimental work[21] are probably due to neglect of the binding energy and momentum of the parent material in the derivation (cf. Ross and Kirkpatrick[22] and treatment of the photoelectric effect below). The velocity of the recoil electron relative to the speed of light is given by

$$\beta = 2(\alpha + \alpha^2)\cos\phi/[(1+\alpha^2) + \alpha^2\cos^2\phi]\,, \qquad (2)$$

where $\alpha = h\nu_0/m_e c^2$, m_e is the rest mass of the electron, ν_0 is the frequency of the incident light, θ is the angle the scattered light makes with the direction of the incident light, and ϕ is the angle of the scattered electron with respect to the same direction. [See Fig. 1 and Table I.] These Compton effect equations are usually derived under the assumption of a compact-photon elastic scattering model, but it has been pointed out that "The photon-ball analogy ... breaks down as soon as further questions are asked ... and it will not help us compute the scattering cross section, which requires the consideration of some of the electromagnetic and quantum-mechanical properties of the electron and the photon."[23]

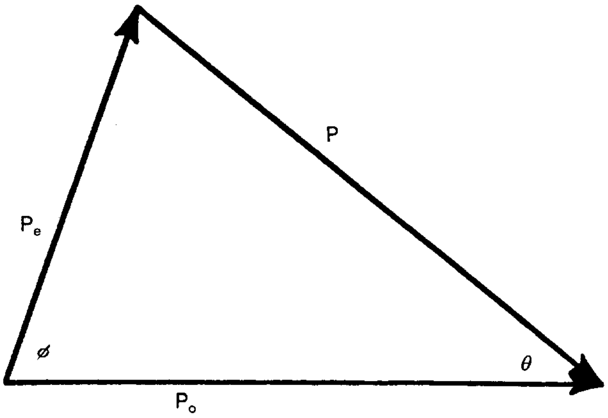

Fig. 1. Compton effect momentum diagram for $h\nu_0 = 3.28\times10^4$ eV and $\phi = 70°$ (a possible but not most probable value), showing initial photon momentum p_0, ejected electron momentum p_e, and final photon momentum p.

It is noteworthy that neither the Compton shift nor the recoil velocity are quantized in the sense of bound electron energy levels. The first is a continuous function of scattering angle and the second is a continuous function of recoil angle and of incoming frequency. For small values of α, the electron goes off approximately in the direction of the electric vector of the incident light and Compton scattering merges smoothly into classical Thomson scattering in the limit of low frequency.

The decreased frequency of the scattered radiation can be accounted for in terms of a Doppler shift, and semiclassical derivations on this basis were suggested by Schrödinger[24] and Halpern,[25] as well as by Compton himself.[26] Debye's derivation,[27] based upon the assumption of "needle radiation" (*Nadelstrahlung,*) appeared almost simultaneously with the Compton version based upon the photon.[28] Additional semiclassical derivations, which accept the quantized interaction but continue to assume a classical, unquantized field, have been provided by a number of more recent writers,[29,30] including the present ones.[31] (For a particularly thorough treatment, see Dodd.[32,33])

The photoelectric effect has received less analysis and merits a more extended treatment. It is usually considered as simply an interaction between a photon and an electron, but this cannot really be correct.

Table I. Compton effect.

$p_0 = 1.75\times10^{-23}$kg m/s
$p_e = 1.13\times10^{-23}$kg m/s at $\phi = 70°$
$p = 1.73\times10^{-23}$kg m/s at $\theta = 37.8°$
$h\nu_0 = 3.28\times10^4$ eV

Einstein's photoelectric equation,

$$m_e c^2[(1/\sqrt{1-\beta^2})-1]\leqslant h\nu_0 - w\,, \tag{3}$$

where w is the work function, the left-hand term is the maximum kinetic energy, and $T_e \rightarrow m_e v^2/2$ in the limit of low velocities is familiar to every student of physics in its nonrelativistic form, and plainly indicates that T_e is a continuous function of ν_0. It follows from Eq. (3) that

$$\beta=\sqrt{2\alpha_w+\alpha_w^2}/(1+\alpha_w)\,, \tag{4}$$

where $\alpha_w=(h\nu_0-w)/m_e c^2$. The work function is not quite constant, being the difference between the Fermi energies of the electrons and the boundary potential of the metal (in a crystal), and exhibits a slight decrease with temperature.[34]

The low quantum efficiency, which is the ratio of photoelectrons to incident light energy divided by $h\nu_0$, on the order of 10^{-3} for most metals, has been ascribed to their high reflectivity.[35] The most probable angle between the direction of the electron motion and the incident light is given by

$$\langle\phi\rangle=\arccos\beta\,, \tag{5}$$

with a cosine-squared distribution around the plane of the electric vector.[36] Like the Compton recoil electron, the photoelectron is ejected in the direction of the incident light's electric vector at low energies and ϕ decreases with the transfer of momentum.[37] This behavior, which is most clearly observed in photoejection from gases, at one time led to the proposal of a transverse vector nature for the photon, since "it is not a scalar bundle of energy but has rather definite vector properties. For it explodes, so to speak, at right angles to its direction of motion."[38]

Taken in isolation, Eq. (3) is somewhat misleading since it suggests that the photoelectric effect involves merely the photon and the electron. However, conservation laws and the expression $p=\sqrt{T^2+2Tm_0c^2}/c$, where p, T, and m_0 represent generalized momenta, kinetic energy, and rest mass, require that an additional mass carry off some of the momentum for total absorption of the photon. Then, just as in the case of the usual Compton effect derivation, we have momentum equations,

$$P_x=h\nu_0/c-m_e\beta c\cos\phi/\sqrt{1-\beta^2} \tag{6a}$$

and

$$P_y=m_e\beta c\sin\phi/\sqrt{1-\beta^2}\,, \tag{6b}$$

where P_x and P_y refer to momentum components of the parent atom(s) having a mass M. (See Fig. 2, which is the same as Fig. 1, except that θ is now redefined as the recoil direction of M, with $\theta=\arctan P_y/P_x$, and Table II.) Since $m:_e/M\approx 0$, the kinetic energy of the parent material is infinitesimally small and drops out of Eq. (3), but its momentum is significant.

An expression of interest is the ratio of electron to incident photon momentum, $r=p_e/p_0$,

$$r=\sqrt{(h\nu_0-w)^2+2m_ec^2(h\nu_0-w)}/h\nu_0 \tag{7}$$

and has a maximum at

$$h\nu_0=w(2m_ec^2-w)/(m_ec^2-w)\approx 2w\,. \tag{8}$$

For this value, $\beta\ll 1, \phi\approx\pi/2$, and classical expressions provide the same answers as relativistic ones. Potassium, here taken to have a work function of 2.25 eV, has $r=337$, a typical maximum value, indicating that the photon momentum is the least significant factor in this energy region. Another expression of interest, the ratio of the forward component of electron momentum to photon momentum, p_{ex}/p_0, goes to its very broad maximum of 2 at about $h\nu_0=676w$ (cf. Compton[39]), while $r\rightarrow 1$ and $P\rightarrow 0$ as $h\nu_0\rightarrow\infty$. Clearly, the photoelectric effect is not a two-body problem. The parent crystal is evidently an essential participant in the interaction between light and the electrons and perhaps mediates it.

It is not necessary to assume the photon in explaining the photoelectric effect. In 1914, Richardson[40] derived Einstein's equation using a thermodynamic argument that considered photoemission as analogous to evaporation from a liquid surface and the work function comparable to

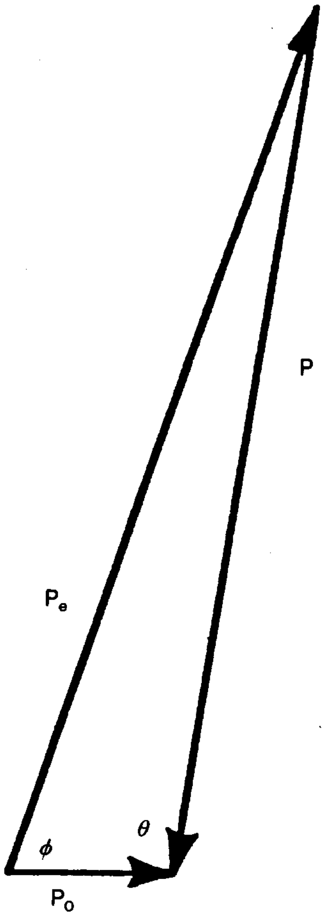

Fig. 2 Photoelectric effect momentum diagram (to different scale) for $h\nu_0=3.28\times10^4$ eV, $w=2.25$ eV, and $\phi=70°$ (most probable value), showing initial photon momentum p_0, ejected electron momentum p_e, and parent material momentum P. For these values, the ratio of electron momentum to incident photon momentum is 5.7.

Table II. Photoelectric effect.

$p_0=1.75\times10^{-23}$ kg m/s
$p_e=9.94\times10^{-23}$ kg m/s at $\phi=70°$
$P=9.48\times10^{-23}$ kg m/s at $\theta=100°$
$h\nu_0=3.28\times10^4$ eV
$w=2.25$ eV
$r=5.68$

a latent heat of vaporization.[41] Richardson later specified that his conclusions were reached "without making any definite hypothesis about the structure of the radiation"[42] in order to avoid the "restricted and doubtful [photon] hypothesis used by Einstein."[43] Curiously, Richardson's theory, although it accounted for all the main features of the phenomenon, appears to have had no impact on the subsequent development of quantum theory, possibly because it seemed to afford no insight into the detailed interactions of radiation and matter. If so, this constitutes one of the ironies in the history of science (cf. Wheaton[44]), since modern investigations have shown that the phenomenon is at least partly a bulk effect in which the photoelectron originates at considerable depths within the crystal and percolates to the surface with substantial energy losses,[45] in closer correspondence with Richardson's theory than with Einstein's. Richardson subsequently went on to win the Nobel Prize in 1928 for his related work in thermionic emission.

In 1927, Wentzel derived Einstein's photoelectric equation and the angular distribution of the electrons on the assumption that the electron could be described by the Schrödinger equation and that the radiation could be treated as an unquantized electromagnetic field.[46] A number of recent semiclassical derivations also exist.[47–49] These have been criticized on the grounds that a zero delay time would violate conservation of energy,[50] but possibly these objections rest on unnecessary assumptions.[51] As Einstein once pointed out, Maxwell's equations refer only to average values.[52]

Whatever the model chosen, there is agreement that an energy $h\nu_0 \gg w$ must be involved in the photoemission, but the microscopic details of the process have never been directly observed and are largely a matter of conjecture. All that is really known is that somewhere an emitter acts as a source of radiated energy and that, ultimately, the photoelectron acts as a sink for this energy. For example, in his analysis of the photoelectric effect in the first decade of this century, Lorentz pictured the electron as a mathematical point whose geometric cross section was irrelevant, since it was coupled with electromagnetic radiation through its electrostatic field.[53] {In a related connection, we might cite Wheaton's criticism of the "simpleminded (and misleading) textbook calculation of the delay time, [which] by assuming that all the [classical wave] energy incident on the cross section of the atom accumulates on an electron, produces the result $\tau = 8\times 10^4$ s!"}.[54]

In any case, it has been pointed out by modern writers that the Einstein equation, lack of time delay, and correct rate of emission all follow from orthodox time-dependent perturbation theory and assumption of a classical electromagnetic wave of frequency ν_0, along the lines of Wentzel, and that this result is implied in standard treatments (e.g., Merzbacher.[55])

A few other items should be briefly mentioned. Contrary to elementary textbook treatments, the photon hypothesis is not required in the derivation of the Bohr atom, and, as mentioned earlier, Bohr absolutely rejected the photon for a decade following publication of his theory. Nor is the photon requisite for derivation of the equation for blackbody radiation since, as Wien pointed out in 1913, it can also be based on wave assumptions.[56] It was also asserted in 1915 by Ehrenfest and Onnes that Planck's formula "*cannot be interpreted in the sense of Einstein's independent light quanta*" [italics theirs].[57] (Ten years later, this insight was confirmed by the development of Bose–Einstein statistics.) In 1917, Einstein derived an expression for Brownian motion based upon the assumption that quanta possessed a directed momentum given by $p = h\nu_0/c$.[58] However, Breit subsequently pointed out that it does not necessarily follow, as Einstein implied, that the success of the derivation proves the correctness (in the sense of uniqueness) of the assumption, and he provided a classical derivation for the same expression.[59] Zero point fluctuations are also said to take place in the absence of [real] quanta.[60] Finally, dispersion theory, which frequently is considered a triumph of quantum mechanics, is largely a reinterpretation of the damping term and oscillator strengths in the classical Lorentz–Drude theory[61] and deals mainly with phenomena that are not describable in terms of quantized energy levels.

Probably the most persuasive evidence one can experience for the existence of the photon consists of watching individual quanta being registered in a scintillation counter or seeing a diffraction pattern build up grain by grain on a photographic film.[62] It is indeed difficult to keep in mind the fact that such phenomena are not certain evidence for the granular structure of light, since, if quantized transfer of energy and momentum to bound matter be granted, all material detectors must be binary in nature[63] even should radiation actually consist, e.g., of classical unquantized fields.

IV. HISTORIC MODELS OF THE PHOTON

Scully and Sargent have stressed the necessity for a "logically consistent definition of the word 'photon'—a statement far more necessary than one might think, for so many contradictory uses exist of this elusive beast."[64] Historically, the photon has indeed received at least four major interpretations, the first three of which we will attempt to describe in this section.

A. The particle model

Elementary survey course textbooks usually leave the impression that the photon is a small, spherical entity along the lines suggested by Lewis and is the ultimate constituent of light in much the same way as the electron is the ultimate constituent of electric current. (Hereafter, we will refer to this purely corpuscular model as Photon I.) What then are supposed to be its properties, and how consistent are these with the physical requirements of such a model?

The interactions of light with matter are consistent with an ability to transfer an energy of $h\nu_0$ and a momentum of $h\nu_0/c$ to bound electrons and with a photon spin of $1\,[\hbar]$. Equivalently, the photon is polarized, in order to account for this property in light observed macroscopically. Photon I is usually supposed to have zero rest mass in order that the relativistic energy expression should have a finite value at $v = c$, although suggestions to the contrary have been made. (Feynman's usage[65] is more of a mathematical device to avoid divergence than a physical attribution.) Indirect measurements indicate that any such mass must be less than 8×10^{-46} kg.[66] A kind of kinematic effective mass analog can be defined for the photon by the relation $m_{\text{photon}} = h\nu_0/c^2$, but if this is taken to be the same as the gravitational mass, it accounts for the gravitational red shift but leads to a deflection by stars only half as great as that predicted by general relativity.[67]

The photon is said to be localized within a volume of λ_0^3

by thermodynamic considerations,[68] where $\lambda_0 = c/\nu_0$ in vacuum. A minimum volume of $\Delta x\ \Delta y\ \Delta z = h^3/\Delta p_x\ \Delta p_y\ \Delta p_z$ following from the uncertainty principle has led to denial that the photon can be considered a point particle.[69] Photon density and transport are usually considered statistically, since the photon is said not to be localizable. A "photon Schrödinger equation," valid for eigenstates, has been used to treat elastic scattering,[70] but the photon's transverse position remains unspecified in the infinite plane-wave representation (cf. Park[71] and Cook[72]).

Returning to elementary textbook treatments, a simplistic billiard ball model of the photon suffices to account for the angular relationship and the transfer of momentum in specular reflection. But the model leads to disconcerting conclusions about ordinary refraction. For, if it is assumed that the tangential component of the velocity is unchanged, elementary vector analysis together with Snell's law (or alternatively, wave optics and de Broglie's momentum relation) require that the photon's velocity in an optical medium must be equal to nc, like that of Newton's corpuscles, where n is the index of refraction, $n \geqslant 1$ ordinary.

If, on the other hand, it is assumed that the photon maintains the ordinary phase velocity of light, $v_w = c/n$, then a different set of unusual properties emerges. Taking the photon as having a "mass" $m_0 = h\nu_0/c^2$, an energy $E_0 = m_0c^2$, and a momentum $p_0 = m_0c$ in vacuum, then in an optical medium, assuming the usual relationships continue to hold, $p = np_0$, $E = n^2E_0$, $m = n^2m_0$, and the model forfeits its appealing simplicity (cf. Arnaud[73]). (For an elementary text that does discuss these problems, see Michels *et al.*[74]) It is difficult to reconcile the foregoing with standard optical theory, which predicts[75]

$$v_g = v_w\left(1 + \frac{\lambda_0}{n}\frac{dn}{d\lambda_0}\right)^{-1} \qquad (9)$$

and leads to a group velocity that is only a few percent lower than the phase velocity for optical frequencies in glass. It is also sometimes hypothesized that photons always travel at c in optical media, but with a short time delay (on the order of 10^{-20} s) between absorption and reemission in order to account for the retardation,[76,77] in apparent contradiction of the assertion of (theoretical) zero time delay in the Compton effect.[78]

As early as 1909, Lorentz had raised the objection that, despite some striking successes of the model, "one cannot speak of propagating lightquanta concentrated in small regions of space that at the same time remain undivided."[79] He pointed out that coherence length in interference experiments (see below) required a longitudinal extension of about a meter for the photon, while ordinary optical properties demanded a lateral extension on the order of the diameter of the telescope for light from distant stars.

Historically, interference has been the most difficult phenomenon for this photon model to account for. As has been known since 1909, multislit interference patterns are a "single-photon effect" equally capable of being produced by short, high-intensity exposures or by very prolonged low-intensity exposures where the average energy in the system at any one time is $\leqslant h\nu_0$.[80] Yet, a two-slit pattern, for example, is completely different from the superposition of the two alternate single-slit patterns. It is as if the photon somehow contrives to pass through both slits and interfere with itself, exactly like a classical wave, in accordance with Dirac's insistence that "the photon interferes only with itself."[81] The attempt to reconcile this behavior with a simple particulate nature has been described by Feynman as being "impossible, *absolutely* impossible, to explain in any classical way, and ... has in it the heart of quantum mechanics. In reality, it contains the *only* mystery" [italics his].[82] Conceivably, the "mystery" is not in the behavior of light but in the retention of the model.

Dirac's dictum that photons never interfere with each other has apparently been contradicted by the demonstration of interference between two laser beams, unless we care to give an unusual definition to the concept of a single photon.[83,84] In this connection, we might recall that in 1922, before becoming famous for the effect that bears his name, Compton had raised the possibility of splitting a photon at the half-silvered mirror of a Michaelson interferometer.[85] Many subsequent writers have claimed that the photon always goes in one direction or the other, but recent experiments appear to show that the photon can indeed be split.[86]

B. The singularity model

For all of the above reasons, Photon I is not an adequate model. This was known in the early decades of the century to physicists who rejected corpuscular quanta. But it may come as a surprise to some readers that Einstein himself was dissatisfied with the model and attempted for many years to improve upon it.

Einstein is usually considered the originator of the doctrine of duality for light[87] (for a differing view, see Hendry),[88] and there is no doubt that he became convinced of the inescapable reality of quanta and the necessity for duality. However, his version of duality was a kind of synthesis, and a dichotomic duality like the later one of Bohr was repugnant to him.[89]

Thus, as early as 1909, he had asserted that "wave structure and quantum structure...are not to be considered as mutually incompatible," and that "the next phase in the development of theoretical physics will bring us to a theory of light that can be interpreted as a kind of fusion of the wave and the emission theory."[90] Apparently he had in mind a derivation of the lightquantum as a singularity in a nonlinear generalization of the electromagnetic wave equation, but this was not successful at that time.

In 1918, he wrote, "I do not doubt any more the *reality* of the radiation quanta" [italics his][91] and also, "If we could only clarify...the principal reasons behind the quanta."[92] In the following year, he published a theoretical derivation of the quanta in the framework of a modified electromagnetic theory, which he considered flawed in that the solutions were not unique.

In 1921, Einstein complained to Ehrenfest that the problem of quanta was enough to drive him to the madhouse.[93] Two years later he suggested in an article that the quanta might be derived from overdetermining the field equations, but this approach also proved disappointing. Around 1925, he was toying with the idea of a "guiding field" (*Fungrungsfelder*) for photons, or, as he ironically phrased it to Bohr, a "ghost field" (*Gespensterfeld*),[94] but he published nothing on the subject. His 1927 paper with Grommer was successful in demonstrating that quanta could be treated as mathematical singularities in the electromagnetic field, but Einstein was dissatisfied with the arbitrariness of the singularities and, thereafter, published no more theoretical investigations of the quanta. However, the problem apparently continued to trouble him. In 1951, four years before his death, he wrote to one of his oldest friends, "All these fifty years of conscious brooding have brought me no closer to the answer to the question, 'What are light quanta?' Of course, today, every rascal thinks he knows the answer, but he is mistaken."[95]

In 1922, Oseen had shown that Maxwell's equations possessed solutions approximating the quanta, and in 1926 de Broglie developed his theory of the double solution, in which the photon was considered as a singularity in the wave, but none of these unitary theories—henceforth to be termed Photon II–appears to have had a significant impact on the physics community.

Bohr's duality could be classified as the alternate use of Photon I and the classical wave model, just as de Broglie's 1927 particle-and-pilot-wave theory, when applied to light, could be loosely considered as their simultaneous employment. A more widely accepted version of the composite model, first formally proposed by Slater in 1924, is that only the corpuscle is "real," while the electromagnetic field serves the same function as Born's ψ field in merely determining probability of appearance of the particle.[96] (A thorough recent exposition has been given by Mayants.[97]) This interpretation, which is found in some textbooks, has been disputed by those who insist that the electric field itself is real and has measurable effects of its own (cf. March[98]). The disagreement is not readily resolvable, since it depends on *a priori* assumptions about not-directly-observable models responsible for the same phenomena, which can only be evaluated on the basis of further implications.

C. Wave packet model

Some elementary quantum texts today consider the photon as a wave packet. Unfortunately, compactness approximating that of a particle would require almost an infinite range of frequencies, and if the usual optical properties apply, such a packet would be expected to disintegrate immediately (except for transitory recombinations) in any dispersive medium, including air.

Elementary tests also frequently treat photons as simple classical wavetrains. If so, they would be subject to the same disability as the wave packet, although less stringently. This model would be logically related to the coherence time (transition lifetime) τ_c and the coherence length $c\tau_c$ of the emitting atoms. Here, τ_c ranges from about 10^{-9} to 10^{-7} s, leading to a typical coherence length on the order of a meter. If this is taken as the extent of the wavetrain, one is reminded more of the transitory "needle radiation" interpretation of the 1920s more than anything suggestive of a compact particle. This model has a certain appealing plausibility, but, unfortunately, large fluctuations (photons?) have been observed in much less than τ_c for chaotic light,[99] while photoemission has been experimentally produced by short chopped segments of such wavetrains.[100] (Hereafter, the wave packet/wavetrain models will be referred to as Photon III.)

It has been suggested that the photon is actually a soliton (which is something like Photon II), and hence immune to dispersion. This possibility has not been disproved, but we have not seen any evidence that would allow us to evaluate it further at this time. Thus none of the influential historic models of the photon appear to withstand critical analysis.

V. THE QED MODEL AND SEMICLASSICAL DERIVATIONS

The final model of the photon to be considered in this article is that provided by quantum electrodynamics, developed by Dirac, Jordan, Pauli, Heisenberg, and Fermi between 1927 and 1931, and henceforth to be referred to as the modern photon or Photon IV. At the outset, it may be said that QED has as much in common with classical electromagnetic theory as with the old quantum theory.[23] Jordan and later de Kronig were said to have developed an interpretation of QED "treating the photon as a mere appearance and not as a genuine unit."[101] In any case, it has been asserted that, although "light, as far as energy and momentum are concerned, cannot be distinguished from a system of particles [with energy $h\nu_0$ and momentum $h\nu_0/c$], in other respects there is no equivalence ... it proves to be impossible to coordinate a real position to the light quantum, and generally we may state that the interpretation of the light quanta as real particles would lead to consequences that are positively wrong"[102] under QED (cf., Heitler[103]). Another writer remarks that "photons really ... are not particles like baseballs or shot ... photons are more like coefficients in a Fourier's series—or increments to the coefficients."[104]

It has also been said that "interference of independent lasers...is not puzzling if we recall that the fringes are described by...the fact that the photon is a quantized excitation of the normal modes of the *entire* system" [italics ours] and that "these functions are the same for both classical and quantum fields. Hence, there is no need to switch from quantum to classical descriptions or to introduce a mysterious wave–particle dualism in order to explain interference and diffraction."[105] (Also see Park.[106]) As Fermi remarked, "We may conclude that the results of the quantum theory of radiation describe this phenomenon in exactly the same way as the classical theory of interference."[107]

This brings us to one of the most striking and provocative features of analysis of interactions between light and matter: So-called semiclassical or neoclassical approaches have been more successful than any of the historic models of the photon in withstanding criticism and simply explaining most—but not all—of the phenomena (for a discussion of the varieties of semiclassical theories, see Jaynes[108] and Senitsky[109]).

Thus there are semiclassical derivations or rederivations of varying success for Bremsstrahlung,[110] the Raman effect,[111] the Klein–Nishina equation, the Lamb shift, spontaneous emission, absorption and stimulated emission, vacuum polarization, the blackbody radiation spectrum,[112] resonance fluorescence, laser operation,[113] "photon echoes,"[114,115] and "photon bunching,"[116] in addition to those for the Compton effect and the photoelectric effect already mentioned.

On the other hand, the scattering of light by light, as predicted by quantum electrodynamics, cannot be derived from semiclassical theory, even in the low-energy limit, without the assumption of nonlinear interactions,[117] and it has also been claimed that no semiclassical theory can account for the total blocking of cascade photons by crossed polarizers[118] (cf. Milonni[119]).

Recently, it has been shown that semiclassical theory constitutes an important special case of QED for coherent states, or, more generally, for quantum states that become deterministic (i.e., nonstatistical) in the classical lim-

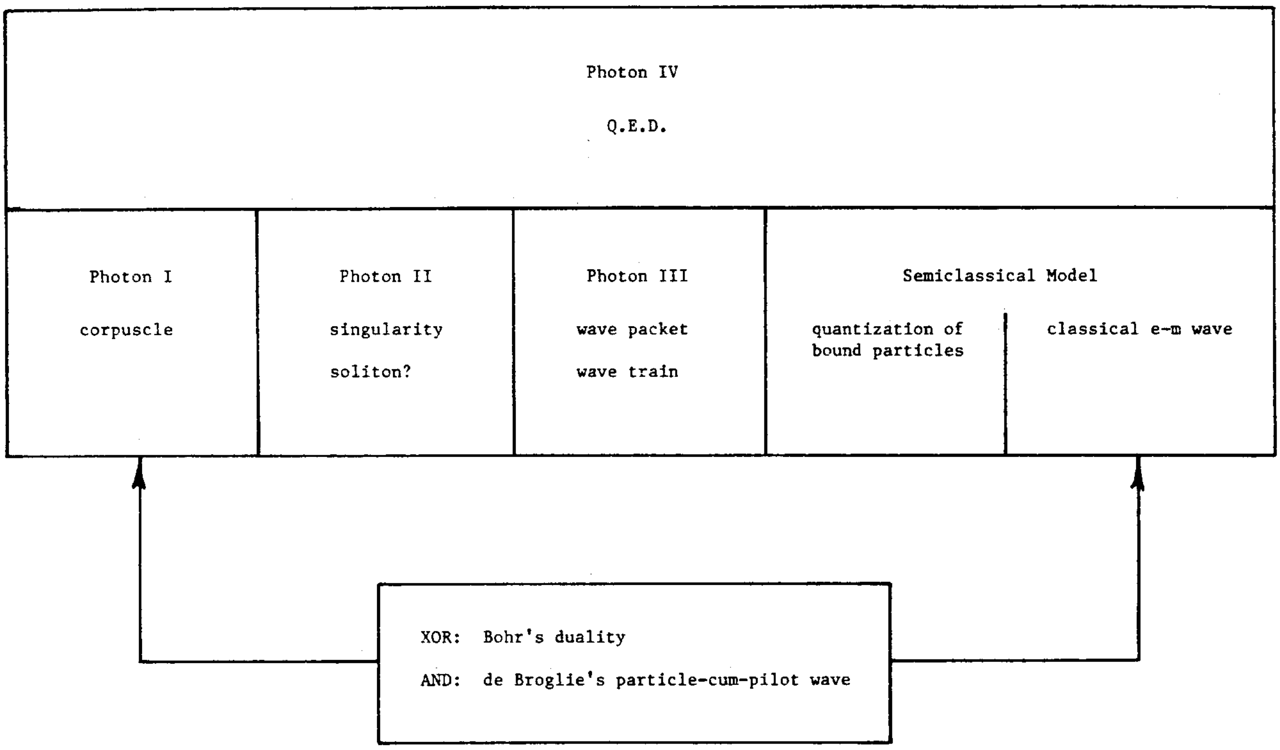

Fig. 3. Relation between models discussed in this article. Each of the pre-QED models accounted for portions of the observed phenomena, whereas the QED model predicts all known phenomena.

it[120,121] (see also Mandel[122]). Thus all of the earlier models have been superseded by QED, but the semiclassical model usually remains the most satisfactory approximation to it, unless Photon II should become better established. (For a diagram of the relationships between models, see Fig. 3.)

VI. CONCLUSION

As we have seen, the term "photon" has meant very different things to different writers. This very ambiguity may have been useful in the first quarter of the century, since it allowed the term to function as a kind of shorthand for transcending the classical limitation of the spreading (i.e., nondirectional) wave without falling into controversy about details, and to deal with an energy and momentum transfer comparable to that between classical particles. But a valuable metaphor seems to have been improperly taken as an identity.

Historically, Photon I displaced the spreading wave in much the same way as the caloric[123] displaced phlogiston.[124] But it has been said that "the fuzzy ball picture of a photon often leads to unnecessary confusion."[105] In today's terms, it is an anachronistic concept, and one that can appear paradoxical if applied literally to phenomena better analyzed in terms of waves. In this regard, Jaynes has castigated "the standard verbal misconceptions [of real QED] (i.e., the buckshot theory of light, which has propagated through several generations of elementary textbooks) with which we brainwash our elementary students,"[84] while Bunge remarks that QED is a field theory containing no hypothesis about the corpuscular nature of photons. The optical duality is then a relic of the 1905–1927 interregnum, a remains serving mainly to mislead students into believing that light is at the same time undulatory and nonundulatory."[125]

We believe these comments refer to an underlying misunderstanding about the role of models that merits further discussion. (The writers apologize for restating the obvious in the interest of clarity.) On the microscopic level, physical reality cannot be directly perceived by the senses and so is not intrinsically knowable in the same sense as macroscopic reality. It can only be approached inferentially by studying measurements registered by macroscopic instruments. Physical models based upon macroscopic experience provide suggestive analogies and may, in certain cases, serve as successively closer approximations to the underlying microscopic reality even though they are usually simplified versions of the entities they are intended to represent. Mathematical models, in turn, are often suggested by physical models (and sometimes vice-versa) and usually represent a further simplification of the latter for mathematical convenience. A mathematical model can often be associated with more than one physical interpretation, as several famous equations can testify. And finally, all models, and mathematical models especially, are subject to the natural selection provided by internal consistency and by experimental tests of their inferences beyond the phenomena they were originally devised to explain.

Thus Einstein's photoelectric equation is essentially a mathematical model that eliminates unnecessary details about structure and extent of the radiation and focuses solely on energy transfer. As such, it is quite reasonable and useful. The mathematical model in turn suggested the localized photon model, but it is a serious error to forget that this was only a hypothesis that might or might not have utility in investigating other phenomena. As a matter of fact, its limited applicability gave rise to the need for duality in radiation models. In turn, duality does no harm if it is confined to limiting cases of mathematical models, but it introduces an unnecessary and misleading element of mysticism into physics if it is naively supposed that somehow light is literally both a particle and a wave. This is not much of a problem to working physicists, who, as Heisenberg pointed out, are conditioned by the evolved [and somewhat analogous or nonrepresentational] usage of the terms,[126] but it can cause serious difficulties to students.

Eighty years ago, the young iconoclast Einstein[52] first proposed the photon hypothesis to a skeptical physics community. If Einstein were reborn today, is it not conceivable that he might now boldly propose that the old quantum theory photon is no less contradictory than the ether and should no longer be given lip service? QED essentially provides quantization of the interaction without invoking the corpuscular model.[127]

In the opinion of the writers, elementary texts would do well to drop the corpuscular photon (Photon I) except as a historical topic on the same level as that of the Bohr atom and to substitute overtly the more widely applicable semiclassical model as a first approximation to the modern QED version (cf. Strnd[128]). The wavetrain model continues to have illustrative value so long as localization is deemphasized. And all of us might bear in mind that beginning students are unlikely to possess much sophistication about levels of models or the realms of applicability of different models and that it may often be necessary for us to point out explicitly the appropriate distinctions.

[a] The authors have attempted to provide a reasonable survey of a voluminous and contradictory literature extending over most of the present century but would approciate having relevant papers they may have overlooked called to their attention.

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The stretched harmonic oscillator: An analytical test of semiclassical approximations[a]

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The stretched harmonic oscillator is a potential that can be varied continuously from the harmonic oscillator to the infinite square well. The corresponding solutions of the one-dimensional Schrödinger equation are almost as simple as those of the usual harmonic oscillator. It is used to study analytically the reliability of semiclassical approximations.

I. INTRODUCTION

Semiclassical approximations are not only a means of relating classical and quantal descriptions of physical problems but they also provide, in many cases, very accurate approximations to the quantal results. However, it is not easy to produce simple yet nontrivial examples that illustrate this point. For this, it is desirable that both the quantal